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Would weighted-student funding enhance intra-district equity in Texas? A simulation using DEA

Shawna Grosskopf[1], Kathy Hayes[2], Lori Taylor[3] and William L Weber[4,*]

[1]*Oregon State University, Corvallis, OR;* [2]*Southern Methodist University, Dallas, TX;* [3]*Texas A&M University, College Station, TX; and* [4]*Southeast Missouri State University, Cape Girardeau, USA*

We use data envelopment analysis to model the educational production function, and then explore how a shift to weighted student funding using the student weights embedded in the Texas School Finance Formula would alter the allocation of inputs and potential outputs. School outputs are measured as value-added reading and math scores on standard achievement tests. We find that if school districts allocated their resources efficiently, then they would not allocate their resources to campuses according to the funding model weights. Policies that promote greater efficiency would also enhance equity in educational outcomes.

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1. Introduction

Researchers have long recognized that the cost of providing a quality education differs from one school district to another. Such differences could arise from differences in labour cost, differences in student need, or economies of scale or scope (Hanushek, 1993).

Policymakers have responded to evidence of differences in cost with school funding formulas that provide additional resources to school districts with higher labour cost (as in Texas, Wyoming, or Florida), higher student needs (as in all states but Nevada, Montana and South Dakota), or a lack of economies of scale (as in Texas, Louisiana or Kansas).[1]

Arguably, the factors that drive differences in cost for school districts also drive differences in cost at the school level. Therefore, researchers and policy makers have become increasingly interested in the distribution of resources within school districts (eg, Miles and Roza, 2006; Ladd, 2008, and Baker, 2011). Weighted student funding (also known as student based budgeting) has arisen as a popular strategy for addressing equity concerns within school districts. Under weighted student funding, resources are allocated within school districts according to a formula based on the numbers and types of students enrolled in each school (Miles and Roza, 2006).

In this paper, we simulate the efficiency and equity effects of a move towards weighted-student funding as a means of determining school budgets. Unlike researchers who examined equity in school resources, we focus on equity in school outcomes, which we measure as value-added test scores in reading and mathematics, controlling for differences in school input prices and the fixed inputs that schools use in the production of value-added test scores.

We examine 2709 schools residing in 175 school districts in three Texas metropolitan areas: Houston, Dallas and San Antonio. We use DEA (data envelopment analysis) to model the multi-output production process of schools. Schools that maximize the various outputs given inputs are efficient and can only expand outputs if given larger amounts of inputs. We compare school output efficiency when schools take inputs as given with school output efficiency when each school can choose the relative amounts of inputs subjected to a fixed budget. Holding the budget constant, some schools might find that they can further expand outputs if they were to reallocate inputs; for instance, by increasing or decreasing teachers and staff relative to non-personnel inputs, such as computer software expenditures.

We also simulate the effects of a change in school budgets towards weighted-student funding. Under weighted-student funding some schools would gain resources and some schools would lose resources. We use DEA to simulate the potential outputs that could be produced under weighted-student funding and compare those outputs with the *status quo*.

While many researchers have used DEA to measure the efficiency of schools *ex post*, a feature of our paper is we use DEA to simulate the effects of a policy change to weighted-student funding *ex ante*. The exercise of comparing the efficiency of schools *ex post* with the efficiency of schools

**Correspondence: William L Weber, Department of Economics and Finance, Southeast Missouri State University, One University Plaza, Cape Girardeau MO 63701, USA.*

[1]For a more complete description of school funding formulas see Verstegen and Jordan (2009), which is the source of the information about formula weights for student need. See Baker and Duncombe (2004) for a discussion of scale adjustments. See Taylor and Fowler (2006) for a discussion of formula adjustments for higher labour cost.

ex ante allows us to simulate how inequality in school outcomes might change under such a policy move. Several indicators of inequality for actual outputs, potential outputs if schools were efficient, and potential outputs that could be produced under weighted-student funding are examined. The next section provides an overview of past research on school efficiency and equity. In the section 'Method' we present our method of measuring school efficiency. The section 'Data' is broken into several subsections describing the outputs, inputs, environmental variables that influence the production technology, and how weighted-student funding would affect school budgets. The section 'Performance estimates' reports the *ex post* estimates of school efficiency under the *status quo* and the *ex ante* estimates of potential school outputs under weighted student funding. The section 'The effects of weighted student funding on inequality' continues the *ex post*/*ex ante* analysis by examining inequality in school outcomes. The final section summarizes and offers some policy implications.

2. Efficiency and equity issues in schools

Many researchers have applied the tools of efficiency analysis to public education. Indeed, one of the first applications of DEA examined 167 elementary schools in the Houston Independent School District (Bessent *et al*, 1982). Numerous studies of educational efficiency have since been undertaken studying individual students (eg, Borge and Naper, 2006; Waldo, 2007; Cherchye *et al*, 2010; Perelman and Santin, 2011, and De Witte and Kortelainen, 2013). Other researchers have examined schools (eg, Mancebon and Mar-Molinero, 2000; Olivera and Santos, 2005; Cordero-Ferrera *et al*, 2008; Cordero-Ferrera *et al*, 2010, and Gronberg *et al*, 2012), school districts (eg, Ruggiero, 2000; Chakraborty *et al*, 2001, and Ouellette and Vierstraete, 2010), the efficiency of public *versus* private schools (eg, Mancebon and Muniz, 2008) and international comparisons of school systems (Gimenez *et al*, 2007). Coates and Lamdin (2002) provide a good exposition of DEA for school administrators and policymakers not well versed in its use.

In addition to technical inefficiency—too few outputs produced from too many inputs—several studies have also considered allocative inefficiency that arises because of an inappropriate choice of inputs or outputs (eg, Haelermans *et al*, 2012, and Haelermans and Ruggiero, 2013). Both studies conclude that efficiency losses because of technical inefficiency are larger than those from allocative inefficiency.

In a series of papers studying Texas school districts Grosskopf *et al* (1997, 1999, 2001) accounted for school district differences in input prices and students' own human capital and measured district technical efficiency in the production of value added on a battery of achievement tests in mathematics, reading, and writing. Grosskopf *et al* (1997) found that policy reforms aimed at equalizing budgets between school districts would generate a distribution of student achievement that exhibited greater inequality than the *status quo*. However, Grosskopf *et al* (1999) found that student achievement gains could be enhanced by relaxing various regulations governing input use. In addition, Grosskopf *et al* (2001) found that greater competition and citizen monitoring can enhance efficiency in school outcomes.

An even larger group of researchers have examined horizontal and vertical equity in education. Horizontal equity refers to the equal treatment of equals (Berne and Stiefel, 1999; Ladd, 2008). Common measures of horizontal equity include the Gini coefficient, range, McLoone index, mean absolute deviation, and coefficient of variation (Toutkoushian and Michael, 2007). In contrast, vertical equity refers to the unequal treatment of students in different circumstances. Different outcomes can be a consequence of different administrative and pedagogical processes used to provide education. In addition, student needs, demographics, socio-economic characteristics, geographical cost of living differences, and local capacity can lead to differences in per pupil spending that might still be regarded as equitable (Murray *et al*, 1998; Ruggiero *et al*, 2002; Taylor, 2006; Cherchye *et al*, 2010).

Rice (2004) argued that the equity and efficiency movements both failed to achieve their goals and that linking the two goals by recognizing their interrelations might provide a more reasonable policy goal. For instance, if schools with-more-difficult-to educate students are to receive greater amounts of resources, the increased funding should be contingent on ensuring the efficient use of those new resources.

Weighted-student funding incorporates all educational and student needs into a formula that drives funding. Students with different needs are weighted differently. Common categories include the number of students in special education, poverty, limited English proficiency, vocational education, grade level, and gifted education. In theory, the formula would be derived from a cost analysis with the amount of funding depending on the specific needs of the students that the school serves but ultimately the political process plays an important role in determining the weights (Ladd, 2008). In practice, weighted-student funding as examined by Miles and Roza (2006) appears to be based on linear cost adjustment factors and ignores any potential interaction between outputs and the various categories, the level and mix of outputs, differences in input prices across schools, or resources from central administration for things like professional development or special programme staff.

Complicating a move to weighted-student funding is accounting for incentive effects associated with different student weights. Cullen (2003) found that school administrators in Texas were more likely to classify students as having a learning disability when the school funding formula provided a greater weight for students with a disability. Using estimates derived from 1991–1992 to 1996–1997 Texas school districts Cullen found that a 10% increase in revenue generated by a special education student led to a 2.1% increase in the student disability rate. The results were most pronounced for learning disabilities that involved subjective judgments on the part of those people evaluating students. Furthermore, there was a greater tendency to classify students as disabled in school districts with a small number of campuses, which enabled greater centralized decision making.

3. Method

Although past researchers have focused on equality in school budgets (eg, Berne and Stiefel, 1994; Miles and Roza, 2006; Toutkoushian and Michael, 2007; Baker, 2011, 2012), our goal in this paper is to focus on equity in school outcomes. Specifically, we want to examine equity in actual reading and math test scores under the *status quo* allocation of resources *versus* the equity outcomes that might occur with a move towards weighted-student funding. Since numerous papers have found various levels of inefficiency in schools; we also want to control for any potential inefficiency that might be present.

A cost function can be used to model a multi-output and multi-input school production process with any variation between minimum costs and actual costs attributed to inefficiency (Ruggiero *et al*, 2002). However, with weighted-student funding schools would receive a budget and rather than minimize the cost of producing a given output, citizens, parents, and policymakers would instead like administrators to try and maximize outputs. As mentioned by Rice (2004), schools that receive larger resources should be expected to make efficient use of those resources. In addition, Ladd (2008) argues that weighted-student funding 'enhances equity defined in terms of outcomes' as long as 'the weights correctly reflect differential needs' (Ladd, 2008, p 416). Therefore, we want our method to assess whether weighted-student funding as determined through the political process has the potential to enhance equity in school outcomes.

We measure school performance by using distance functions. Our performance indicator builds on the directional output distance function developed by Chambers *et al* (1996, 1998). This distance function is an outgrowth of Luenberger's (1992) benefit function that was used in consumer theory. Directional distance functions can be estimated using a linear programming method called DEA that was developed by Charnes *et al* (1978).

We illustrate our method of measuring efficiency and simulating a policy change to weighted-student funding graphically. A particular school within a school district uses variable inputs, $x \in R^N_+$, and fixed inputs, $F \in R^J_+$, to produce outputs, $y \in R^M_+$. Each school uses a technology that transforms the inputs into outputs that we represent by the output possibility set: $P(x, F) = \{y:(x, F)$ can produce $y\}$. In our analysis of Texas schools we assume that the variable inputs (x) consist of school specific personnel (teachers) and non-personnel (maintenance) inputs. The fixed inputs (F) include a share of the central administration overhead expenses and the socio-economic and demographic characteristics of the student population.

Suppose the school faces input prices $w \in R^N_+$ with which to hire inputs (x). The cost of using x and F to produce y is $\sum w_n x_n \leqslant c$. The school has discretion over variable input use as long as it satisfies the budget constraint. Different choices of x will generate different output possibility sets. Figure 1 depicts the budget constraint facing the school district and the output possibility sets for three choices of variable inputs, x^A, x^B, and x^*. We let the set of outputs that can be produced given fixed inputs and the budget (c) be represented by the budget-constrained output possibility set: $IP(w/c, F) = \{y:(x, F)$ can produce y and $\sum w_n x_n \leqslant c\}$. In DEA form this set can be written as

$$IP\left(\frac{w_o}{c_o}, F_o\right) = \Bigg\{ y : \sum_{k=1}^{K} \lambda_k y_{km}, \geqslant y_m,\ m = 1, \dots, M,$$

$$\sum_{k=1}^{K} \lambda_k x_{kn} \leqslant x_n,\ n = 1, \dots, N,$$

$$\sum_{k=1}^{K} \lambda_k F_{kj} \leqslant F_{oj},\ j = 1, \dots, J, \sum_{n=1}^{N} w_{on} x_n \leqslant c_o,$$

$$\lambda_k \geqslant 0,\ k = 1, \dots, K \Bigg\}. (1)$$

Each of the individual production possibility sets, $P(x, F)$, is a subset of the budget-constrained production possibility set, $IP(w/c, F)$. Inputs are efficiently allocated when, given the budget and input prices, the school is able to produce the maximum amounts of the two outputs. In Figure 1, the largest production possibility set occurs when the chosen inputs are x^*. Other choices of inputs, say x^A or x^B, are affordable, but yield smaller production possibility sets than x^*. Much of the school choice

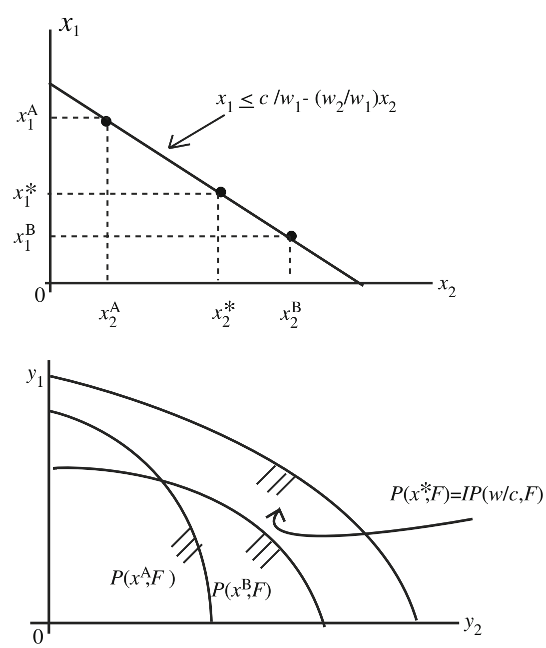

Figure 1 Campus inefficiency.

literature argues that rules and regulations constrain schools in what they can achieve and that giving schools greater discretion over inputs is one way to enhance school efficiency.

The two production possibility sets have various properties. First, when the school has access to more inputs, (x, F), the set $P(x, F)$ expands. Second, when the school faces lower input prices or has a larger budget or is endowed with a larger amount of fixed input, the set $IP(w/c, F)$ expands. We make use of these properties in our simulation exercise that examines a policy of weighted-student funding.

We measure efficiency using the directional output distance function. Given inputs, the directional output distance function finds the maximum expansion in the various outputs that could be produced if a school were efficient. Outputs are expanded for the directional vector $g = (g_1, \ldots, g_M)$. Formally, we can write this distance function as

$$\vec{D}_o(x, F, y; g) = \max_{\beta}\{\beta : (y + \beta g) \in P(x, F)\}. \qquad (2)$$

To estimate the directional distance function we choose a directional vector of $g = (1, 1, \ldots, 1)$ so that the $\vec{D}_o(x, F, y; g)$ gives the maximum unit expansion in each of the M outputs.

When a school is efficient, $\vec{D}_o(x, F, y; g) = 0$ meaning that it is not possible to further expand outputs given inputs. Inefficient schools have $\vec{D}_o(x, F, y; g) > 0$ with larger values indicating greater inefficiency. Figure 2 illustrates how $\vec{D}_o(x, F, y; g)$ is estimated given $g = (1, 1)$ and two outputs: y_1 = value-added on a reading test and y_2 = value-added on a mathematics test.

Given $P(x, F)$ we observe a particular school (campus) within a school district to produce at point A. The function $\vec{D}_o(x, F, y; g)$ gives the maximum expansion in the two outputs that is feasible given the technology. If campus A were to use its resources efficiently it could produce at B on the frontier of

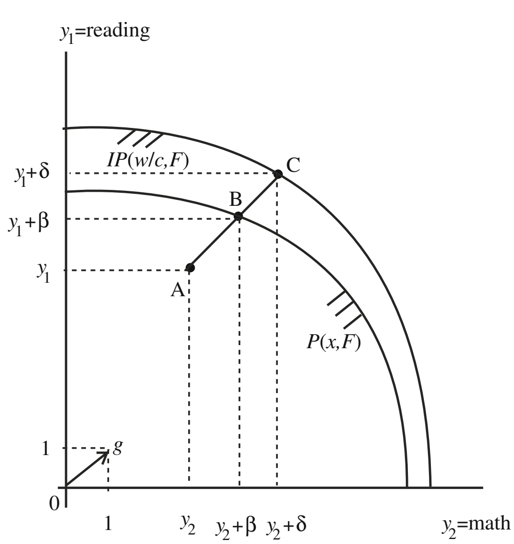

Figure 2 School inefficiency.

$P(x, F)$. Campus A's reading score could expand from y_1 to $y_1 + \beta$ and their math score could increase from y_2 to $y_2 + \beta$.

We estimate $\vec{D}_o(x, F, y; g)$ for school 'o' using DEA as

$$\vec{D}_o(x_o, F_o, y_o; 1) = \max_{z,\beta}\Bigg\{\beta : \sum_{k=1}^{K} \lambda_k y_{km} \geqslant y_{om} + \beta,$$

$$m = 1, \ldots, M, \sum_{k=1}^{K} \lambda_k x_{kn} \leqslant x_{on}, \quad n = 1, \ldots, N,$$

$$\sum_{k=1}^{K} \lambda_k F_{kj} \leqslant F_{oj}, j = 1, \ldots, J, \lambda_k \geqslant 0, k = 1, \ldots, K\Bigg\}. \quad (3)$$

On the right-hand side of (3) are the observed outputs and inputs of school 'o' and on the left-hand side of (3) is the best-practice DEA technology comprising linear combinations of all observed schools' outputs and inputs.

To measure efficiency relative to $IP(w/c, F)$ we use the budget-constrained directional output distance function. Again, this function seeks the maximum expansion in outputs for the directional vector g, but in this case the school can reallocate inputs (x) as long as the choice of inputs satisfy the budget constraint. This distance function takes the form

$$I\vec{D}_o(w/c, F, y; g) = \max_{\delta, x}\{\delta : (y + \delta g) \in IP(w/c, F)\} \qquad (4)$$

and is illustrated in Figure 2. Holding inputs constant, school A can increase output to B by reducing technical inefficiency. However, if school A were able to optimally reallocate their inputs it could increase output even further: reading and math scores could expand to $y_1 + \delta$ and $y_2 + \delta$ at point C. A campus produces on the frontier of $IP(w/c, F)$ if $\delta = 0$ and is inefficient if $\delta > 0$.

There might be some campuses that produce outputs such that $\beta = 0$ but $\delta > 0$. These schools are technically efficient but could expand outputs by reallocating their inputs. There might be other schools that are both technically efficient, $\beta = 0$, and have also allocated inputs efficiently, $\delta = 0$. For the schools with $\beta = \delta = 0$ outputs can be expanded only if the school receives a larger budget.

One of our objectives is to simulate how outputs could change if each school received a budget that had been determined by weighted-student funding. Schools that receive more inputs would see their production possibilities expand, while schools that receive fewer inputs would see their production possibilities contract. Let x_{wsf} equal the inputs a school would receive under weighted-student funding. Let $c_{wsf} = wx_{wsf}$ equal the budget the school would receive under weighted-student funding.

How will potential math and reading scores change under weighted-student funding? If $x_{wsf} \geqslant x$, then the *status quo* and weighted-student funding production possibility sets are such that $P(x, F) \subseteq P(x_{wsf}, F)$ and potential outputs can expand. On the other hand, if $x_{wsf} \leqslant x$ potential outputs will contract under weighted-student funding. Similarly, if $c_{wsf} \geqslant c$, the status quo and weighted-student funding budget-constrained production possibility sets are such that $IP(w/c, F) \subseteq IP(w/c_{wsf}, F)$. In contrast, if $c_{wsf} \leqslant c$, the new budget-constrained production possibility set will be no larger than the *status quo* budget-constrained production possibility set.

Figure 3 illustrates two possible shifts in $IP(w/c, F)$ with a move to weighted-student funding. We observe a school operating inefficiently at point A. If that school were efficient it could expand outputs to point C. If school A receives a larger budget ($c < c_{wsf}$) under weighted student funding then $IP(w/c_{wsf}, F)$ shifts towards the northeast and it could expand outputs to point E if it were to use those resources efficiently. In contrast, if school A receives a smaller budget $IP(w/c_{wsf}, F)$ would shift toward the southwest and the school would only be able to produce at point D.

We anticipate winners and losers in a move towards weighted-student funding. If the schools that lose money ($c > c_{wsf}$) are inefficient, then the reduction in output will be tempered by the existing inefficiency. If the schools that gain money ($c < c_{wsf}$) are efficient under the *status quo*, then the expansion in the budget has real potential to help those schools achieve greater outputs. In contrast, if the schools that lose money are efficient under the *status quo*, then the reduction in the budget will cause schools outputs to fall. Likewise, if the schools that gain money are inefficient under the *status quo*, then the expansion in the budget, while increasing potential outputs, will have no guarantee of increasing actual outputs.

The budget-constrained directional distance function for school 'o' with directional vector $g = (1, \ldots, 1)$ takes the form:

$$\vec{ID}_o\left(\frac{w_o}{c_o}, F_o, y_o; 1\right) = \max_{z,\delta,x}\left\{\delta : \sum_{k=1}^{K}\lambda_k y_{km} \geqslant y_{om} + \delta,\ m = 1,\ldots,M,\right.$$

$$\sum_{k=1}^{K}\lambda_k x_{kn} \leqslant x_n,\ n = 1,\ldots,N,$$

$$\sum_{k=1}^{K}\lambda_k F_{kj} \leqslant F_{oj},\ j = 1,\ldots,J,$$

$$\left.\sum_{n=1}^{N} w_{on} x_n \leqslant c_o, \lambda_k \geqslant 0,\ k = 1,\ldots,K\right\}. \qquad (5)$$

4. Data

The data for our analysis come from the Texas Education Agency (TEA) and cover the three largest metropolitan areas in Texas—Dallas, Houston and San Antonio—during the 2008–2009 school year. These three metropolitan areas were chosen because they are the largest in Texas and among the largest in the nation. Nearly half of the public school students in Texas reside in one of these three metropolitan areas. In addition, the Houston Independent School District currently uses a form of weighted-student funding in allocating funds to each school within the district although their weighted-student funding formula is not perfectly consistent with the state's school funding formula. Our analysis includes all schools in traditional public school districts with complete data that were located in one of the three metropolitan areas.[2]

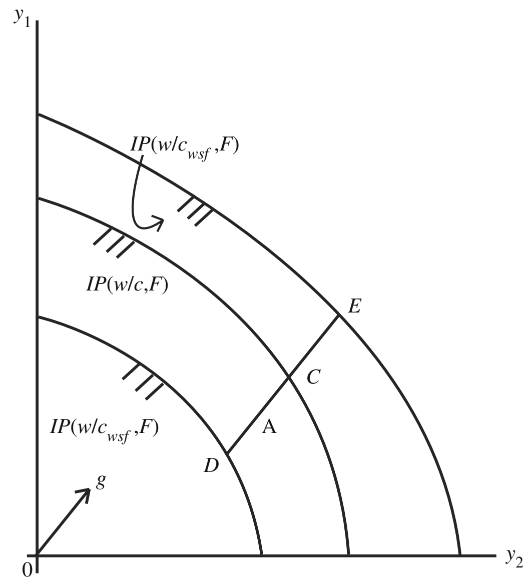

Figure 3 Output gains or losses under weighted-student funding.

4.1. Outputs

The two outputs schools produce are based on the Texas Assessment of Knowledge and Skills (TAKS)—a group of high stakes tests administered every year from 2003 to 2012. Student performance on TAKS was used not only for federal accountability under the No Child Left Behind Act of 2001, but also for state accountability purposes. Students in the third and eighth grades had to pass TAKS to be promoted to the next grade, and students in the 11th grade had to pass TAKS in order to graduate. TAKS tests in mathematics and reading/language arts were administered annually in grades 3–11. Tests in other subjects such as science and history were also administered, but not in every grade level.

Following Borland and Howsen (1992), Duckworth and Seligman (2006), Reback (2008), Gronberg *et al* (2012), and Grosskopf *et al* (2015) we measure school outputs as the normalized gain score in reading and mathematics. An early paper discussing the assumptions and problems in aggregating student achievement scores to school averages is by Spencer (1983). The normalizations we use are designed to address concerns about reversion to the mean found in traditional gain scores.

[2]Because they have access to a different educational technology, open enrolment charter schools have been excluded.

We use test scores for student (i), grade (g), at time or year (t), denoted as S_{igt}. We measure each student's performance in each subject (reading or math) relative to the performance of all other students in the state at the same grade level with same past score:

$$Y_{igt} = \frac{S_{igt} - E\left(S_{igt} \middle| S_{t,g-1,t-1}\right)}{\left[E\left(S_{igt}^2 \middle| S_{i,g-1,t-1}\right) - E\left(S_{igt} \middle| S_{i,g-1,t-1}\right)^2\right]^{0.5}} \tag{6}$$

In calculating Y_{igt} we calculate the average test score at time t, grade g, for students scoring $S_{i,g-1,t-1}$ at time $t-1$, in grade $g-1$. For example, we divide all fifth-grade students in the state into groups or bins based on their fourth-grade math test scores in 2008. We then calculate the average fifth grade math score and the standard deviation (the denominator of (6)) of the average fifth grade math score for each bin. The average fifth-grade math score for each bin is the expected score for students in the bin. Our variable Y_{igt} measures the number of standard deviations from the expected score. This is a type of z-score, which has a mean of zero. These z-scores are averaged over all the students in each school to arrive at a school level measure of reading and math value added.[3]

Because outputs with negative values are not tractable for analysis purposes, we further transform the z-scores into normal curve equivalent (NCE) scores. The normal curve equivalent, which is a monotonic transformation commonly used in the education literature, is defined as $50 + 21.06 \times z$. We multiply the NCE by the number of students at the school to obtain the aggregate school outputs.

4.2. Inputs

We use data on school and district expenditures to measure the campus-specific resources and the central administration resources in each school district. Under the state's Public Education Information Management System, school districts are required to report the fund, function, object and financial unit (campus) for each dollar they spend, using standard definitions published by TEA. We use these data to calculate the level of personnel and non-personnel expenditures allocated to each campus. Expenditures not allocated to a specific campus are treated as overhead.

All measures (personnel expenditures at the campus level, non-personnel expenditures at the campus level, central administration personnel expenditures and central administration non-personnel expenditures) are aggregate amounts at the particular campus, but exclude food and student transportation expenditures.

The expenditure variables used include all operating expenditures regardless of the sources of revenue: direct salary expenditures, contributions to the pension system, group health and life insurance, and other outlays for employee benefits. Personnel expenditures include payments for contract workers as well as salaried employees. Non-personnel expenditures includes payments for rent, utilities, and supplies.

Previous research has found that there are substantial inter-district differences in labour costs in Texas. Therefore, transforming the personnel and non-personnel expenditures into effective input quantities requires use of a labour cost index. Following Gronberg *et al* (2011) and Grosskopf *et al* (2015), we estimate a hedonic wage model wherein teacher salaries are a function of teacher demographics and cost factors that are outside of school district control.[4] The hedonic model predicts the wage each school would have to pay to hire a teacher with zero years of experience and a bachelor's degree, holding all other teacher characteristics constant at the statewide mean and suppressing any charter school differentials. The effective quantity of school personnel equals school expenditure on personnel, divided by the wage. This approach treats compensation as a direct indicator of educator quality and is consistent with work by Loeb and Page (2000).

There is no such evidence to suggest that there are systematic differences in the cost of non-personnel inputs. Therefore, we presume that the cost of non-personnel inputs is constant throughout the three metropolitan areas, and normalize its price to one.

4.3. Other environmental factors

The model includes indicators for several environmental factors that influence the educational technology but which are not purchased inputs.[5] To capture variations in costs that derive from variations in student needs, we include the percentages of students in each district who have high English proficiency (%HEP), percentage non-special education students (%non-specialed) and per cent with high socio-economic status (%HighSES). Finally, we include the number of students on a particular campus as a fixed input for that campus, so that students are not reallocated (bussed) across campuses, say from one primary campus to another primary campus in the same school district.

4.4. Weighted student funding simulation

Under the Texas school funding model some students generate additional revenues for a school district. For example, an economically disadvantaged student will generate 20% more revenue than a student who is not economically disadvantaged. A student who is in bilingual education programmes would generate 10% more than a student who is not in bilingual education. Furthermore, the weights are additive, meaning that

[3]Students for whom the prior test score was missing are treated as one of the groups. This is equivalent to assuming that all students with missing pre-test data had the state average pre-test score.

[4]Details of this model are described in an Appendix that can be found at http://cstl-hcb.semo.edu/bweber.

[5]Ruggiero *et al* (2002) provide an alternative method of incorporating environmental variables into DEA.

a student who is both economically disadvantaged and has low English proficiency would generate 30% more revenue than a student who is neither. Funding model weights are provided for students in compensatory education programs for economically disadvantaged students, special education programs, bilingual education programs, gifted education programme, career and technology education programs and the high school programme.

In Texas, school districts are not required to rely on the state's funding formula weights for internal allocation purposes. For example, the Houston Independent School District uses lower weights for compensatory and special education students and uses weights for homeless and refugee students that have no direct parallel in the state's funding formula (HISD, 2014).[6] Our simulation compares the level of performance given the *status quo* budget with the level of performance a school might achieve if each district allocated resources to schools according the state's school funding model, assuming no change in overhead expenses. Let $s=1, \ldots, S$ represent a particular school (campus) within the district and let $i=1, \ldots, P$ represent the number of programmes. To obtain the amount school s would receive under weighted-student funding we calculate the school share of weighted-average daily attendance ($SWADA_s$) based on the school's programmatic enrolment:

$$SWADA_s = \frac{\sum_{i=1}^{P} DPP_i \times STUD_{si}}{\sum_{s=1}^{S}\sum_{i=1}^{P} DPP_i \times STUD_{si}} \tag{7}$$

where DPP_i is the district revenue per pupil for programme i, and $STUD_{si}$ is enrolment at school s in programme i. We then apply the school share of district weighted-average daily attendance, $SWADA_s$, to the district total spending on campus personnel and non-personnel to yield the school-level weighted-student funding. We note that DPP_i varies across districts because of adjustments for district size, cost differences, and regulations such as hold-harmless provisions.

5. Performance estimates

The data are taken from the Texas Education Agency for the 2008–2009 school year for 175 school districts in the Dallas (70 districts), Houston (66 districts), and San Antonio (39 districts) metropolitan areas and includes 387 high schools, 618 middle schools, 1694 elementary schools, and 10 mixed schools. Students in grades 3–11 take the Texas Assessment of Knowledge and Skills, a standardized achievement test. Each school produces value added on a statewide reading achievement test (y_1) and value added on a statewide mathematics achievement test (y_2) using personnel (x_1) and non-personnel expenditures (x_2). The personnel input is measured as the number of teacher units at the school. In addition, two kinds of school district central administration overhead expenses are allocated to each campus within the school district on a per pupil basis: central administration core operating overhead expenses (F_1) and central administration overhead payroll expenses (F_2). We also control for the number of students at each school (F_3), the per cent of students at the school who have high English proficiency (F_4), the per cent of students at the school who are deemed high socio-economic status (F_5), and the per cent of students at the school who are not special education students (F_6).

Table 1 reports descriptive statistics. The average school spends (wx) slightly more than US$5 million on the two variable inputs of personnel and non-personnel maintenance and utilities and has an additional $1.36 million ($653 thousand and $483 thousand) in central administration overhead spending. Among the 802 average students, approximately 81% have high English proficiency and 91% are non-special education students, but only 44% are deemed high socio-economic status. Average reading and math NCE scores per pupil are 50.8 and 50.7. The wide range in aggregate school outputs—from 112.7 to 223 181 for reading and 220 to 212 686 for math scores—are not necessarily outliers but are because of the wide range in the number of students since average value-added scores derived in Equation (6) are summed over all students. DeWitte and Marques (2010) describe one method for addressing outliers in DEA.

We solve four different distance functions using DEA for each school (campus) within the district for the directional vector $g=(1, 1)$. In model 1 we estimate $\vec{D}_o(x, F, y; 1)$ for each school and obtain an estimate of β, which gives the simultaneous expansion in reading and math test scores given variable inputs (x) and fixed inputs (F). In model 2 we estimate $\overrightarrow{ID}_o(w/c, F, y; 1)$ for each school and obtain an estimate of δ. In models 3 and 4, we switch to a system of weighted-student funding and simulate the change in reading and math scores that could result if schools were efficient. In model 3 each school receives variable inputs corresponding to our weighted-student funding formula (x_{wsf}) holding the fixed inputs (F) of central administration overhead and student socio-economic and demographic characteristics constant. In model 4, each school receives a budget that is consistent with weighted-student funding (c_{wsf}) holding input prices (w) and fixed inputs (F) constant.

Our model allows the different types of schools—high schools, middle schools, elementary schools, and mixed schools—to face different technologies. That is, a high school and an elementary school with identical amounts of variable and fixed inputs can have output possibility sets ($P(x, F)$) that are shaped and positioned differently. Similarly, a high school and an elementary school with the same budget, input prices, and fixed inputs can have indirect output possibility sets ($IP(w/c, F)$) that are shaped and positioned differently.

[6]According to the Resource Allocation Handbook for the fiscal year 2014–2015, published by the Houston Independent School District, the weights used in distributing resources are broken down as follows: Special Education: 0.15, State Compensatory Education (50% free/reduced lunch and 50% at-risk): 0.15, Gifted and Talented: 0.12, Vocational Education (CATE): 0.35, Bilingual/ELL (English Language Learner): 0.10, Homeless: 0.05, Refugee: 0.05.

Table 1 Descriptive statistics for 2709 schools

	Mean	*Standard Deviation*	*Minimum*	*Maximum*
y_1 = read	40 738.5	27 751.5	112.7	223 181
y_1/student	50.8	5.6	4.3	71
y_2 = math	40 705.6	27 944.9	220.0	212 686
y_2/student	50.7	4.4	30.0	77
x_1 = # of personnel	1 149.4	726.6	21.4	6497
x_2 = non-personnel exp.	476 843.8	515 593.7	0.0	5 063 815
F_1 = personnel overhead	653 633.7	560 800.0	1 711.9	8 116 015
F_2 = non-personnel overhead	483 186.8	703 036.7	710.1	28 610 110
F_3 = students	802.1	543.6	3.0	4572
F_4 = % high English prof.	0.81	0.19	0.1	1
F_5 = % high socio-econ. status	0.44	0.30	0.0	1
F_6 = % non-special ed.	0.91	0.04	0.6	1
w_1	3,937.0	109.3	3 427.9	4093
w_2	1.0	1.0	1.0	1
$wx = c$	5 027 680.8	3 345 020.3	73 308.0	30 560 288
wx/student	6 457.9	1 434.1	2 810.8	25 893

Table 2 Potential test score gains from enhanced efficiency under the *status quo* (SQ) and weighted student funding (WSF). $\hat{\beta}$ represents the gain from reducing technical inefficiency and $\hat{\delta}$ represents the gain from reducing technical inefficiency and allocative inefficiency

		All Schools	*Elementary Schools*	*Middle Schools*	*High Schools*	*Mixed Schools*
# of schools		2709	1694	618	387	10
$\hat{\beta}$/student	SQ	7.24	8.81	4.51	4.91	0.74
$\hat{\delta}$/student	SQ	7.98	9.60	5.18	5.55	0.89
$\hat{\beta}$/student	WSF	6.52	8.38	3.97	2.84	−7.78
$\hat{\delta}$/student	WSF	7.26	8.39	4.54	4.79	−4.94

This assumption allows for the possibility that it might be easier or more difficult to educate an elementary school student than a high school student with the same demographic characteristics. However, schools of the same type, say elementary schools, face the same technology regardless of the metropolitan area they reside in.

Although the model estimates derived from (3) and (5) give the aggregate addition to reading and math test scores, for ease of exposition we report the estimates of inefficiency in Table 2 on a per student basis.

Elementary schools have the most technical inefficiency ($\hat{\beta}$). followed by high schools, middle schools, and mixed schools. For elementary schools, the mean estimate of $\hat{\beta} = 8.81$ standardized points per student indicates the amount that reading and math test scores could increase if the average school were to become technically efficient. A similar pattern of inefficiency arises when performance is measured relative to $IP(w/c, F)$. Elementary schools could increase reading and math scores by an additional 0.79 points ($\hat{\delta} - \hat{\beta}$) per student if they could reallocate their existing budgets by choosing the optimal mix of school personnel and non-personnel inputs.

Next we simulate potential inefficiency if districts allocated resources to schools consistent with weighted-student funding using the state's funding formula weights. To do the simulation we first estimate the budget the school would receive as described by Equation (7): c_{wsf}. Then, given c_{wsf} and the actual input prices (w), actual fixed inputs (F), and actual outputs (y), we re-estimate $\overrightarrow{ID}_o(w/c_{wsf}, F, y; 1)$ by substituting c_{wsf} for c in Equation (5). Schools that receive a larger budget will see their production possibility frontiers shift outward and will have $\hat{\delta}_{wsf} > \hat{\delta}$. Schools that receive a smaller budget will see their production possibility frontier shift inward resulting in a contraction in potential outputs, which will show up as a decline in inefficiency. That is, $\hat{\delta}_{wsf} < \hat{\delta}$.

We also estimated the amount of inputs (x) schools would receive under weighted-student funding. Here, we assume that the share of the budget allocated to non-personnel expenditures remains constant: if a school allocated 45% of their actual budget to non-personnel expenditures we allocate 45% of the budget they receive under weighted-student funding to non-personnel expenditures. That is, $share = (w_2x_2)/(c) = (w_2x_{2,wsf})/(c_{wsf})$ so that $x_{2,wsf} = (c_{wsf})/(w_2)$. Given the share allocated to non-personnel expenditures we calculate the quantity of personnel as $x_{1,wsf} = (c_{wsf} - w_2x_{2,wsf})/(w_1)$. We use simulated quantities of the variable inputs, $x_{1,wsf}$ and $x_{2,wsf}$, along with the actual fixed inputs (F) and the actual outputs (y) in calculating $\overrightarrow{D}_o(x_{wsf}, F, y; 1)$ given by Equation (3).

Average technical inefficiency ($\hat{\beta}$) and overall inefficiency ($\hat{\delta}$) are lower for the pooled sample of 2709 schools under weighted-student funding than they are under the *status quo* for each type of school. This finding indicates that on average, a movement toward weighted-student funding will result in the average school receiving fewer resources, which shifts the production possibility sets towards the origin causing potential outputs to shrink.

The DEA estimates yield distributions of inefficiencies that are not normally distributed. Kneip *et al* (2013) show that standard central limit theorems do not hold for means of inefficiency scores estimated via DEA. We test the null hypothesis that various statistics or distribution functions of the estimates of δ for the *status-quo* and for δ under weighted-student funding are equal using Li's (1996) *t*-test and report the results in Table 3. For Li's *t*-test we bootstrap the results 500 times following Li and Racine (2006).[7] Li's *t*-test rejects the null hypothesis of equal distributions of inefficiency under the *status quo* and the simulated policy of weighted-student funding for all school types. We conclude that a move toward weighted-student funding will result in a leftward shift in the empirical distribution functions of inefficiencies, which implies that potential outputs will shrink under a policy of weighted-student funding.

6. The effects of weighted student funding on inequality

Next, we examine various measures of vertical equity for the *status quo* under our simulated policy move towards weighted-student funding. Table 4 reports five different measures of inequality for school resources and outcomes. Brazer's coefficient of variation (CV) equals the inter-quartile range as a proportion of the median and the Gini coefficient ranges from 0 (perfect equality) to 1 (perfect inequality). We also report the Theil inequality index, the range, and the ratio of the 95 percentile value to the 5 percentile value.[8] All variables are measured in per pupil terms. Spending per student exhibits greater inequality than outputs per student: Brazer's CV is approximately 1.5 times greater for spending per student than it is for the reading test score per student and 1.88 times greater than it is for the math test score per student. Similarly, the Gini coefficient is twice as large for spending per student as it is for the reading score per student and 2.6 times larger for spending per student than for the math score per student. We also find greater inequality in the actual level of reading scores (y_1) than in math scores (y_2).

As shown in Table 4 policies that reduce school inefficiency tend to enhance equality. Comparing actual reading scores (y_1) with potential reading scores ($y_1+\beta$) every measure of inequality is reduced if technical inefficiency is reduced and output is expanded. The same is true for math scores except for the CV which increases slightly from 0.113 to 0.114. When comparing actual reading scores with potential reading scores ($y_1+\delta$) if technical efficiency is reduced and if school resources are allocated efficiently we find all measures of inequality are reduced. The same is true for math scores. This pattern suggests that inefficiency is an important source of outcomes' inequality.

[7]Pagan and Ullah (1999) discuss bootstrapping of kernel distributions.

[8]For the random variable x the Gini coefficient is calculated as $G = 1-((2)/(N-1))(N-(\sum_{i=1}^{N} i\times x_i)/(\sum_{i=1}^{N} x_i))$ where i is the rank of x_i. The Theil index is calculated as $T=(1/N)\sum_{i=1}^{N} x_i/\bar{x}\ln x_i/\bar{x}$.

Table 3 Will weighted-student funding change potential outputs? Non-parametric tests

	Elementary	*Middle*	*High Schools*	*Mixed Schools*
Li's *t*-test	3.51	3.89	2.13	1.96
(Prob > *T*)	(0.01)	(0.01)	(0.03)	(0.05)

Table 4 Inequality measures for 2709 schools-all variables per student

Variable	*CV*	*Gini*	*Theil*	*Range*	*95 to 5 pct. Ratio*
wx	0.212	0.139	0.021	23 082	1.76
wx_{wsf}	0.188	0.086	0.012	8429	1.68
		Reading scores			
y_1	0.142	0.061	0.0062	66.70	1.43
$y_1+\beta$	0.134	0.048	0.0046	66.70	1.36
$y_1+\delta$	0.130	0.051	0.0041	46.26	1.34
$y_1+\beta$(wsf)	0.132	0.056	0.0055	48.63	1.40
$y_1+\delta$(wsf)	0.127	0.052	0.0045	45.61	1.36
		Math scores			
y_2	0.113	0.053	0.0037	47.39	1.32
$y_2+\beta$	0.114	0.046	0.0034	38.81	1.31
$y_2+\delta$	0.113	0.048	0.0037	47.38	1.32
$y_2+\beta$(wsf)	0.112	0.050	0.0043	49.95	1.35
$y_2+\delta$(wsf)	0.107	0.045	0.0034	42.38	1.31

Next, we examine inequality in reading and math scores with a move toward weighted-student funding. Comparing actual reading (y_1) and math scores (y_2) with simulated reading ($y_1+\delta_{wsf}$) and math scores ($y_2+\delta_{wsf}$) we see a decline in all five measures of inequality for reading and four out of five inequality measures for math scores if both technical and allocative inefficiencies are reduced.

We further examine the effects of the simulated move towards weighted-student funding in Table 5 and Figure 4. Table 5 reports the number of winners and losers under weighted-student funding. Comparing potential output relative to $IP(w/c, F)$, with potential outputs relative to $IP(w/c_{wsf}, F)$ we see that 668 schools (24.6%) would see their potential outputs expand, 1351 schools (49.9%) would see their potential outputs contract, and 690 schools (25.5%) would see no change in their potential outputs. Elementary schools have both the largest number of schools which would gain under weighted-student funding, 445 (26.3%), but also have the largest number of schools that would lose under weighted-student funding, 944 (55.7%). The average gain for all 2709 schools is 0.81 points on the reading and math tests while the average loss is 1.84 points. For elementary, middle, and high schools the potential gains average between 0.79 to 0.82 points, while the potential loss in the two outputs averages 1.66 for elementary schools, 2.24 for middle schools, and 2.07 for high schools.

Figure 4 plots the estimates of $\overrightarrow{ID}_o(w/c, F, y; 1)$ for the *status quo* budget (vertical axis) against $\overrightarrow{ID}_o(w/c_{wsf}, F, y; 1)$ for the budget under weighted-student funding (horizontal axis). There were 229 schools that produced on the frontier of $IP(w/c, F)$ under the *status quo* and those schools lie along the horizontal axis. Out of those 229 schools 41 are to the right of the origin and under weighted-student funding would receive a larger budget that would allow an increase in value added outputs if they could use the new funds efficiently. The 103 schools to the left of the origin would see their budgets shrink under weighted-student funding and given their initial efficient level of production, the smaller budget would cause a decline in math and reading test scores. The remaining 85 frontier schools would receive the same budget under weighted-student funding. The 2480 schools that are inefficient given the status quo lie above the horizontal axis. We draw a 45^o line as a reference. To the right of the 45^o line lie 627 schools, which would experience an increase in their budget under weighted-student funding. Along the 45^o line (but excluding the origin)

Table 5 Winners and losers under weighted student funding using the Texas' formula weights

		$N=2709$ *All Schools*	$N=1694$ *Elementary*	$N=618$ *Middle*	$N=387$ *High Schools*	$N=10$ *Mixed*
# of winners	$\hat{\beta}<\hat{\beta}_{wsf}$	967	657	251	58	1
# of losers	$\hat{\beta}>\hat{\beta}_{wsf}$	1419	866	267	280	6
# no change	$\hat{\beta}=\hat{\beta}_{wsf}$	323	171	100	49	3
Average gain		1.23	1.28	1.04	1.46	1.28
Average loss		2.12	1.81	2.24	3.16	14.42
# of winners	$\hat{\delta}<\hat{\delta}_{wsf}$	668	445	148	74	1
# of losers	$\hat{\delta}>\hat{\delta}_{wsf}$	1351	944	231	170	6
# no change	$\hat{\delta}=\hat{\delta}_{wsf}$	690	305	239	143	3
Average gain		0.81	0.82	0.81	0.79	1.89
Average loss		1.84	1.66	2.24	2.07	10.02

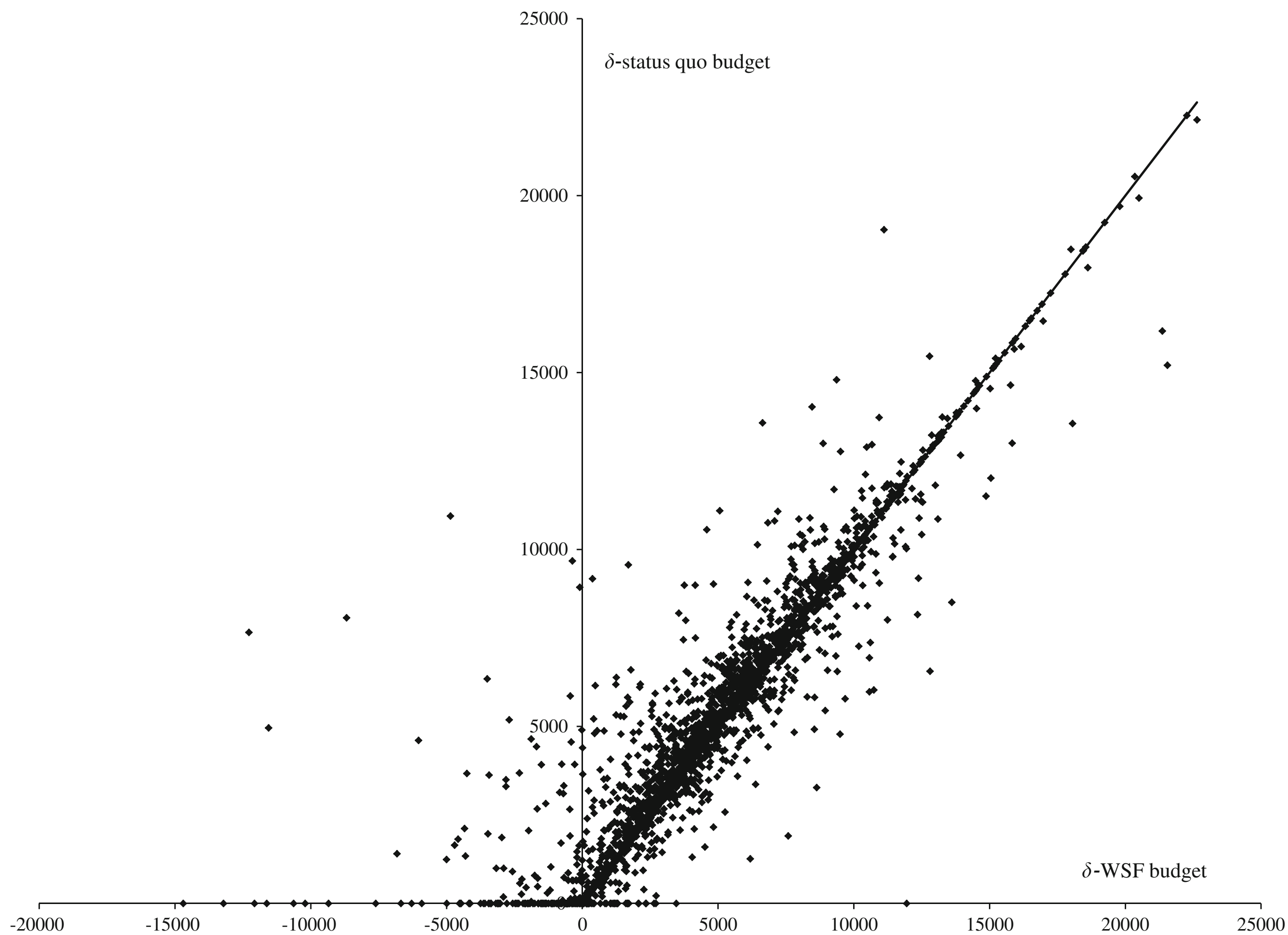

Figure 4 Budget inefficiency under the *status quo versus* weighted-student funding.

lie 605 inefficient schools which would receive the same budget under weighted-student funding as they do under the status quo. To the left of the 45° line lie the remaining 1248 schools, which would receive a smaller budget under weighted-student funding.

Our simulation predicts one thing with relative certainty. To the extent that school resources matter in the production of value added test scores, the efficient schools under the *status quo*, which lie to the left of the origin (103 schools) would see their production possibilities shrink. The 41 efficient schools that receive more inputs and a larger budget under weighted-student funding outputs can possibly expand if the resources are used efficiently. Given that these schools were efficient to begin with, it seems reasonable to think that these schools can efficiently use the extra resources to expand output.

For the 2480 schools that were inefficient under the *status quo* several possibilities emerge with a move to weighted-student funding. First, those schools could remain inefficient and the change towards weighted-student funding only redistributes inputs with no change in value-added test scores. Second, those schools that were inefficient under the *status quo* somehow become efficient under weighted-student funding policy. For the 627 schools that lie to the right of the 45° line a change towards weighted-student funding and greater efficiency would result in higher test scores; this possibility seems much less certain. Consider the inefficient schools that lie between the vertical axis and the 45° line. These 1168 schools would see their production possibilities shrink under weighted-student funding, but they were inefficient for the *status quo* budget. In fact, their inefficiency was great enough so that even though their budgets shrink, enhanced efficiency could more than offset their smaller budgets. Perhaps the declining budgets and resources would refocus administrator and teacher efforts on getting the most out of the now smaller set of inputs. Finally, 80 schools are inefficient given their *status quo* budgets and lie to the left of the vertical axis. These 80 schools would see their production possibilities shrink to the extent that even if they were to become fully efficient they would still experience a decline in value-added test scores.

7. Conclusions

Policymakers have long sought to foster equity in public school funding. Although equalizing per pupil expenditures was once the goal, differences in the marginal cost of educating students has caused researchers to shift their focus towards equalizing school outcomes. One such policy under consideration is weighted-student funding where school funding formulas would take account of the higher costs of educating students with disabilities, students who come from disadvantaged socio-economic backgrounds, or students who do not speak English as their native language.

We simulate a policy move towards weighted-student funding for three Texas metropolitan areas-Dallas, Houston and San Antonio, which together account for approximately half of all students in Texas. Our model compares potential value added on reading and math scores with what might be achieved by the various schools if they were to adopt the best-practice technology from the sample of observed schools. In addition, our simulation indicates that weighted-student funding would generate both winners and losers. Under weighted-student funding using the state's funding weights 668 schools would gain resources and be able to increase value added test scores by 0.81 points if they used those resources efficiently. However, 135 schools would lose resources and be subject to a potential loss of 1.84 points.

Several important findings emerge from our study. First, school resources are more unequally distributed than school outcomes as measured by value added on standardized reading and math achievement tests. Second, although equality could be enhanced by a move to weighted student funding, increases in outcomes' equality could also be enhanced by policies that increase school efficiency. Third, much of our analysis depends on the appropriateness of the formula weights. These weights tend to be chosen via the political process and might not be appropriate for maximizing students' outcomes.

We offer several caveats of our study. First, although our simulation shows that a move to weighted-student funding based on the state's funding model could enhance equity in outcomes as measured by value-added test scores, schools also produce other outputs that we have not accounted for such as socialization, preparation for the job market, and extracurricular activities. Second, changes in the school funding formula would likely provide school administrators an incentive to reclassify some students as having a disability. Third, our simulation showing enhanced equity in educational outcomes is predicated on schools reducing various technical and allocative inefficiencies. If inefficient schools that receive enhanced funding cannot reduce existing inefficiencies the new funding formula will be for nought.

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